Weyl's *Raum-Zeit-Materie* and the Philosophy of Science

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Abstract In this contribution I explore the philosophical underpinning of Weyl's interpretation of Relativity as emerging from *Raum-Zeit-Materie*. I emphasize the important distinction between the philosophical and the mathematical methods, as well as the dichotomy and relationship between time and consciousness. Weyl identified the latter as the conceptual engine moving the whole history of Western philosophy. and the revolutionary relevance of relativity for its representation is investigated together with the conceptual underpinning of Weyl's philosophy of science. In identifying the main traits of Weyl's philosophy of science in 1918, I also offer a philosophical analysis of some underlying concepts of unified field theory.

1 Introduction

It is not an easy task to discuss the philosophical underpinning of Weyl's *Raum-Zeit-Materie* and his interpretation of Einstein's relativity theories. After 100 years from its publication, we have a number of excellent studies that dealt with it,[1] underlining the relevance of Weyl's work for the history and development of gauge theory (see [18, 19]. Therefore, what I will try to offer in my contribution is a novel reconstruction of Weyl's philosophical reflection on the foundations of gauge theory. I shall mostly refer to the first and second edition of *Raum-Zeit-Materie* in order to limit the discussion to very concrete and specific topics that however were and still are of capital importance both in the history and in the epistemology of science. Before

[1] We have nowadays various philosophical approaches to the conceptual analysis of Weyl's work, apart from the pivotal work by Thomas Ryckman, *The Reign of Relativity* that takes into account a transcendental approach and confronts Weyl's view with Husserl's Phenomenology [16]. We also find approaches integrating the historical and philosophical perspective, such as Sieroka's [20], or those focusing on Weyl's group-theoretic approach and based on Ontic Structural Realism, such as French's, Ladyman's and Bueno's. A more recent attempt at reading the philosophical underpinning of Weyl's proposal is constituted by studies on four-dimensionalism and eternalism [15].

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entering the discussion of the text, I would like to draw attention to a passage taken from *Symmetry* (1952) and that became a classic in Weyl's studies:

> Symmetry is a vast subject, significant in art and nature. Mathematics lies at its root, and it would be hard to find a better one on which to demonstrate the working of the mathematical intellect. [26, p. 145]

Now, this passage obviously was commented regarding the definition of symmetry offered therein. This time, however, I would like to emphasize the reference that Weyl makes here to "the working of the mathematical intellect". The mathematical intellect, as we shall see, embodies the metaphor of a specific perspective that through symbolic construction (see [3, 6, 7, 12, 13] leads to the objective understanding of the world. We can say that gauge theory illustrates the power of this mathematical intellect thanks to symmetry that dictates the form of interactions. By means of this epistemological standpoint, major achievements in high-energy physics can be seen as disclosing the connection between art (technology and engineering) and nature. Thus, the interesting aspect emerging from the abovementioned lines is that Weyl does not think of philosophy as a means by which we can build up bridges among sciences or among art and nature. It is mathematics that can do that, symmetry can do that. To say this, however, does not imply the absence of philosophy from any process leading to the understanding of the fundamental link between art and nature, nor its absence from physics.

2 Philosophy, Mathematics and Physics

What is then the exact relationship among philosophy, mathematics and physics that Weyl proposed in 1918? I shall focus my analysis on the *Preface* to *Raum-Zeit-Materie*. In it, we read:

> At the same time it was my wish to present this great subject [Relativity n.d.a] as an illustration of the intermingling of philosophical, mathematical, and physical thought, a study which is dear to my heart. This could be done only by building up the theory systematically from the foundations, and by restricting attention throughout to the principles. But I have not been able to satisfy these self-imposed requirements: the mathematician predominates at the expense of the philosopher. [21 v]

Here Weyl is providing an important hint regarding his view of the philosophical and mathematical approaches. Mathematics and philosophy strongly differ regarding the methodology. Philosophical method is identified with the systematic building up of the theory, and it proceeds only in a deductive way from its foundations. On the contrary, mathematics focuses on the principles of a theory but lacks systematicity, or at least can acquire it a posteriori. However, as we shall see in the next sections, for Weyl this lack of systematic deduction of a theory from its foundations is not something negative. It is just something different. Furthermore, the mathematical approach is what enables us to construct bridges among disciplines and different

branches of science.[2] We could interpret this take as due to the separation of the concept of systematicity, reserved to philosophy and physical theories, from that of architectonics. The latter entails ends, scopes. Mathematics is not just a heuristic tool; mathematics, in Weyl's view, is able to disclose the scope of the structure of the whole universe. In *Raum-Zeit-Materie*, Weyl declares to endorse the mathematical approach. The mathematician inside him prevailed, but he did not disregard philosophy at all. Let us consider the following passage:

> Einstein's Theory of Relativity has advanced our ideas of the structure of the cosmos a step further. It is as if a wall which separated us from Truth has collapsed. Wider expanses and greater depths are now exposed to the searching eye of knowledge, regions of which we had not even a presentiment. It has brought us much nearer to grasping the plan that underlies all physical happening (*welche dem physischen Weltgeschehen innewohnt*).[3] [21 v]

At that time, these words could sound prophetic. We now know that relativity disclosed to us objects that we could not even imagine 100 years ago, such as black holes. Weyl explicitly defines relativity as a revolution, as a cataclysm "which has swept away space, time, and matter hitherto regarded as the firmest pillars of natural science" [21p. 2]. There is something mystical in the way in which he presents Einstein and his theory, which for him can "make place for a view of things of wider scope, and entailing a deeper vision" [21, p. 2]. What can these passages mean? What is the 'Truth' that is now in front of us and that we couldn't see before? In order to understand what Weyl had in mind we have to look at the way in which he presents space, time and matter from a philosophical standpoint. Only in this way we can grasp the meaning of the "fall of the wall" that separated us from truth. Indeed, the notions of spacetime and matter as developed by Einstein's theory change our view of "happening".

3 Space-Time-Matter and the Foundations of All Happening

When talking about the "happening" and the revolutionary way of presenting it offered by relativity, is Weyl referring to objective or subjective happening or both of them? To both and in a very literal way, I argue. Consider the following passage:

> Space and time are commonly regarded as the forms of existence of the real world, matter as its substance. A definite portion of matter occupies a definite part of space at a definite moment of time. It is in the composite idea of motion that these three fundamental conceptions enter

[2] For a mature view on this topic, see [24].

[3] The notion of physical happening is of fundamental relevance here. In 1918-1920 Weyl follows very closely the lines of Husserl's *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie* [11], and I suggest that he gives a meaning to this expression that is very similar, albeit not identical to Husserl's. For Ryckman's take on this, see "The philosophical roots of the gauge principle: Weyl and transcendental phenomenological idealism" [17]. Further studies on Weyl's affinities and differences with Husserl, see [1, 10, 12].

> into intimate relationship. Descartes defined the objective of the exact sciences as consisting in the description of all happening (*alles Geschehen*) in terms of these three fundamental conceptions, thus referring them to motion. [21, p. 1]

What Einstein's relativity accomplished is the unification of subjective and objective happening. From the standpoint of mathematical physics, Weyl's unified field theory was meant to be the expression of such a revolutionary move in history. The revolutionary character of relativity was given by its capacity of putting together subjective and objective happening by means of a new conception of the relationship of geometry and spacetime and the possibility to unify consciousness and external reality into one action:

> As the doer and endurer of actions I become a single individual with a psychical reality attached to a body which has its place in space among the material things of the external world, and by which I am in communication with other similar individuals. Consciousness, without surrendering its immanence, becomes a piece of reality, becomes this particular person, namely myself, who was born and will die. Moreover, as a result of this, consciousness spreads out its web, in the form of time, over reality. [21, p. 6]

In fact what makes it possible to unify all happening within the new framework of relativity is the revolutionary way of presenting time and it is worth spending the next subsection on it.

3.1 Raum-Zeit-Materie and Philosophy of Time

Already at the beginning of the *Introduction* to *Raum-Zeit-Materie*, we find the following definition:

> Time is the primitive form of the stream of consciousness. It is a fact, however obscure and perplexing to our minds, that the contents of consciousness do not present themselves simply as being (such as conceptions, numbers, etc.), but as being now filling the form of the enduring present with a varying content. So that one does not say this is but this is now, yet now no more. If we project ourselves outside the stream of consciousness and represent its content as an object, it becomes an event happening in time, the separate stages of which stand to one another in the relations of earlier and later. [21, p. 5]

In the previous passage Weyl describes the subjective representation of time as a primitive form of the flow or stream of consciousness. For him, only an enduring present exists and with a varying content. The now is the parameter to which one associates the content of consciousness or the subjective experience. However, we are beings projecting outside both the flow and its content and generate by means of this projection an objective representation of this content as something happening in time and according to an order relationship of 'earlier' and 'later'. In other words, relativity shows that what we call cosmic time is nothing else than the result of the projection of an enduring present with varying content. In this way time from a pure

form of the flow of consciousness becomes something pertaining to the objective world.[4]

In *Philosophy of Mathematics and Natural Science,* Weyl further clarified his view:

> The objective world is, it does not happen. Only to the gaze of my consciousness, crawling along the lifeline of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time. [25, p. 116]

It is our specific perception, which includes our bodies that makes us project the flow of time outside onto the external realm. The truth is that we live in an enduring present. About the world, about the universe at its most profound level, we can only say that it is.[5] Weyl pertained to a generation of physicists who thought that relativity describes the world as four-dimensional continuum, but that there was more to be said about other conceptions of the world and that not all happening was absorbed by classical physics.[6] It becomes now clearer the sense in which Weyl thought that his unified field theory could embrace all happening. General Relativity theory changes our conception of space, time, matter and therefore motion, but also sheds new light on the relationship between objectivity and subjectivity. Weyl believes that Einstein's theory can overcome this dualism thereby reshaping our conception of time and consciousness that ultimately is possible by abandoning the dichotomy between form and matter, substance and attributes and by embracing a dynamical view of the interaction of matter and fields.

3.2 **Raum-Zeit-Materie** *and the History of Philosophy*

The topics of time and consciousness are central to the Introduction to *Raum-Zeit-Materie*. This might sound surprising at first, but it is even more surprising the fact that Weyl wants to find the right place of Relativity theories within the history of philosophy, by taking the cue from the discussion of the concepts of time and consciousness and their close relationship. This is clear in the following passage:

> Since the human mind first wakened from slumber, and was allowed to give itself free rein, it has never ceased to feel the profoundly mysterious nature of time-consciousness, of the progression of the world in time, of Becoming. It is one of those ultimate metaphysical problems which philosophy has striven to elucidate and unravel at every stage of its history. The Greeks made Space the subject-matter of a science of supreme simplicity and certainty. Out of it grew, in the mind of classical antiquity, the idea of pure science. Geometry became one of the most powerful expressions of that sovereignty of the intellect that inspired the thought of those times. [21, p. 1]

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[4]Eddington [8], for example, concluded that "consciousness, looking out through a private door, can learn by direct insight an underlying character of the world which physical measurements do not betray".

[5]Weyl here supports what in current philosophy is called the Block Universe view.

[6]For the historical roots of such a conception in Riemann and others, see Boi [4].

The problem of Becoming or the dichotomy between Being and Becoming has been at the roots of the history of philosophy and its development throughout the centuries. The solution that the Greek offered was based on the reification of space by means of geometry and the representation of time as a "moving image of eternity", as Plato's *Timaeus* suggested. With relativity not only space but also time constitute the subject matter of science. Spacetime is therefore the object of the theory and General Relativity describes the dynamics and the mutual active and passive interaction of spacetime and matter, by means of curvature and bending. What is certainly striking in some sense is that Weyl believes that for more than 2000 years philosophy tried to solve the mystery of time flow and consciousness. However, one might question whether our modern idea of consciousness—which is not unambiguously defined even today—was really at the roots of the problem of Becoming. I suggest that Weyl is at least correct in his insight regarding the problem of the flow of time as intrinsically related to that of Becoming. This emerged not only in the pre-socratic tradition, but also in Plato's *Timaeus* as one of the first examples in Western thought of a geometrization of the physical world. It is in that dialogue that time (*chronos*) is portrayed as an image, as something generated and not pertaining to the realm of Forms. However, for Plato, time is physical or at least its representation embodied by the trajectories of planetary motion has physical meaning. Furthermore, the category of "coming into being" is used by Plato to grasp the function of time in view of our knowledge and measurement of the universe. The knowledge of the physical world depends on an operational definition of time, but the latter alone is not fundamental from the standpoint of the mathematical intellect. Time results fundamental only together with space in view of the generation of any physical world. For this reason, I claim, Weyl presents the history of philosophy as marked by attempts at resolving the problem of Becoming, as continuous attempts to explain or justify the relationship between time flow and consciousness that ended up in the reification of space as the subject-matter of natural science. Nobody before Einstein, at least in Weyl's view, had the insight of making spacetime and its geometry the fundamental structure from which the physical world could have been represented and in a way that included a new way of portraying the relationship between time and consciousness. In this sense, Relativity introduced a catastrophic revolution in Western thought.

4 The Foundations of Mathematics, Space and Philosophy

We are now in a position to appreciate the relevance attributed by Weyl to philosophy and its history—an approach that he conserved throughout his life thanks to the influence of his wife Helene and the study of Cassirer's works [5], see also [7]. What is left to be clarified is that with the formulation of relativity and its representation of spacetime, the relevance of philosophy is to be ascribed to its role of investigating the foundations of space:

> Now, if on the one hand it is very satisfactory to be able to give a common ground in the theory of knowledge for the many varieties of statements concerning space, spatial configurations, and spatial relations which, taken together, constitute geometry, it must on the other hand be emphasised that this demonstrates very clearly with what little right mathematics may claim to expose the intuitional nature of space. Geometry contains no trace of that which makes the space of intuition what it is in virtue of its own entirely distinctive qualities which are not shared by "states of addition-machines" and "gas-mixtures" and "systems of solutions of linear equations". [21, p. 26]

This passage clarifies that geometry is necessary in order to grasp the fundamental physical reality, but that only outside of geometry, in the realm of philosophy, we can find the answer to the deeper question "what is space?". This position comes as no surprise, considering that Weyl was sympathetic with intuitionism. Geometry, he believes, cannot show the intimate and ultimate nature of space.[7] The following passage clarifies also which branch of philosophy is necessary to make space comprehensible and this is metaphysics:

> It is left to metaphysics to make this "comprehensible" or indeed to show why and in what sense it is incomprehensible. We as mathematicians have reason to be proud of the wonderful insight into the knowledge of space which we gain, but, at the same time, we must recognise with humility that our conceptual theories enable us to grasp only one aspect of the nature of space, that which, moreover, is most formal and superficial. [21, p. 26]

One question that might rise in reading what Weyl suggests here for space is whether this is also valid for time. This is an open question, at least in *Raum-Zeit-Materie*. What we can certainly appreciate in it is that Weyl thought that philosophy is fundamentally different from natural science and mathematics. However, philosophy is fundamental for GR and GR is fundamental for philosophy: there is a mutual and beneficious interaction among the two. On the one hand, one notices the great advancement produced by GR for the philosophy of time and epistemology; on the other hand, it is thanks to philosophy that the great potentialities of GR are understood. The philosophical standpoint, in Weyl's view, can grasp how GR encompasses both subjectivity and objectivity in the world. In order to reach this higher standpoint, Weyl suggests to reinterpret GR by unifying gauge invariance and covariance of laws that are expression of objectivity with the inclusions of the observations of measurements in constructing a theory of gravitation. And it does so through geometry. General relativity as a universal theory of gravitation offers the possibility of unifying different realms through geometry and ultimately this represents 'the sovereignty of the intellect' but only if inserted within the unified field theory extension that Weyl proposed. In Weyl's view the transition from the special to the general theory of relativity is a purely mathematical process. To formulate physical laws so that they remain covariant for arbitrary transformations is a possibility that is purely mathematical in essence and denotes no peculiarity of these laws.[8] However,

[7] Note that this position is very close to Eddington's view of space and this is the result, I claim, of neo-Kantian readings and of the influence of Brower's intuitionism.

[8] It would be interesting to compare Weyl's view with Einstein's and Kretschmann's. For a reconstruction of the Kretschmann-Einstein debate on the foundations of relativity and the meaning of general covariance, see Norton [14].

a new gauge factor appears when it is assumed that the metrical structure of the world is not given a priori, but that the quadratic form is related to matter by gauge invariant laws. In other words, Weyl believed that his unified field theory was the most genuine expression of the great revolution introduced by Einstein. This is a consequence of Weyl's assumption according to which to overcome any dualism is the goal of scientific enquiry and of relativity in the highest sense:

> The physical world-picture here described in its first outlines is characterised by the dualism of matter and field, between which there is a reciprocal action. Not till the advent of the theory of relativity was this dualism overcome, and, indeed, in favour of a physics based solely on fields. [21, p. 68]

Therefore, Weyl's attempt to find a unified field theory should be read as a strategy to lead relativity to its more radical consequences by following upon the mathematical method and the underlying fundamental concept of the unity of nature.

5 Weyl's Philosophy of Science?

Whereas in *Philosophy of Mathematics and Natural Science*, as well as in the late period of his life, Weyl was more explicit in drawing the main tenets of his reflections on science and philosophy of science, in *Raum-Zeit-Materie* we have to consider bits of the texts in order to reconstruct his views. Since *Raum-Zeit-Materie* is a book on his interpretation of relativity, we find in it traces of Weyl's view of scientific theories and his take on holism that will become more explicit from 1927 onward (see [9]). However, this position is already present in 1918 and 1920. First, let us consider that there are two ways for Weyl in which we can portray a physical theory:

1. As a system (objectively)
2. Through symbolic construction (including the subjective within it), as the result of a "tower" or sequence of theories.[9]

The two views are not in contrast, only dogmatism counts them as opposite. This assumption is fundamental in order to understand the connotation of holism that Weyl suggests:

> We cannot merely test a single law detached from this theoretical fabric! The connection between direct experience and the objective element behind it, which reason seeks to grasp conceptually in a theory, is not so simple that every single statement of the theory has a meaning which may be verified by direct intuition. We shall see more and more clearly in the sequel that Geometry, Mechanics, and Physics form an inseparable theoretical whole in this way. We must never lose sight of this totality when we enquire whether these sciences interpret rationally the reality which proclaims itself in all subjective experiences of consciousness, and which itself transcends consciousness: that is, truth forms a system. [21, p. 67]

[9] For further details on Weyl's view of levels or tower of levels of reality, see [23].

Both mathematical consistency and truth value principles can be seen as what encompasses consciousness within a theory. It is not possible for Weyl to admit the correspondence with reality of a theory by testing each by each its statements. A theory must be considered in its totality. Another important point emerging from the text is the importance for Weyl to make the passage from SR to GR possible by means of mathematics, and geometry in particular, which however is the result of a deeper level of physical reality yet to be penetrated. This belief is what drove Weyl in his early proposal of unified field theory and even if he dropped his idea out of the theory of relativity, yet he conserved this fundamental assumption in the later editions of *Ram-Zeit-Materie*:

> Only the consciousness that passes on in one portion of this world experiences the detached piece which comes to meet it and passes behind it, as history, that is, as a process that is going forward in time and takes place in space. This four-dimensional space is metrical like Euclidean space, but the quadratic form which determines its metrical structure is not definitely positive, but has one negative dimension. This circumstance is certainly of no mathematical importance, but has a deep significance for reality and the relationship of its action. [21, p. 217]

For Weyl, the great advance in our knowledge consists in recognising that the scene of action of reality is "a four-dimensional world, in which space and time are linked together indissolubly" [21, p. 217]. This assumption allows us to crystallize the physical world and offers an overview that excludes the qualitative and intuitive experience of time and space from objective knowledge (even if knowledge and understanding might need intuition at first). How? This is possible by means of symbolic construction, it is thanks to the deep interaction of the philosophical and mathematical methods that the quadratic form determining the four-dimensional spacetime is related to matter by generally invariant laws. This is the result of a fundamental fact of the theory, namely Weyl notices that according to the form of Pythagoras' Theorem, whereas the potential of the electromagnetic field is built up from the coefficients of an invariant linear differential form of the world-coordinates, the potential of the gravitational field is made up of the coefficients of an invariant quadratic differential form.[10] Now, since the potential of the gravitational field can be given in this form, the necessity of this fact is not empirically based, but, as Weyl underlines, it comes from geometry, from what he calls "the observations of measurements", it does not actually derive from the direct observation of gravitational phenomena. It becomes clearer now why he presents his unified field approach by means of analogy:

> The same fact is indispensable if we wish to solve the problem of the relativity of motion; it also enables us to complete the analogy mentioned above, according to which the metrical field is related to matter in the same way as the electric field to electricity. Only if we accept this fact does the theory briefly quoted at the end of the previous section become possible, according to which gravitation is a mode of expression (*Äusserungsweise*) of the metrical field. [21, p. 226]

[10]For further details on this, see Weyl [22] and the recent collection by Bernard and Lobo [2] on Weyl and the problem of space.

From the philosophical standpoint, Weyl is endorsing an epistemology of science privileging the a priori over the empirical data and indirect proofs over direct proofs. The analogy between the gravitational and the electric potential is only indirect and indeed it will be discarded a few months later by Einstein. However, it was advanced by Weyl in the name of the unity and systematicity of scientific theories, a truth value or at least an aim that also Yang and Mills pursued in 1954. What is certainly interesting in the abovementioned passage is the definition of gravitation as a mode of expression of the metrical field. This term "mode of expression" (*Äusserungsweise*) was highly diffused among the German-speaking philosophers, because it was used by both Husserl and Heidegger. A mode of expression, for instance, can be open or closed, considering that Weyl believed in the systematic view of scientific theories and their progressive inclusion of higher standpoints, he probably meant gravitation to be one mode of the metrical field, susceptible of being modified by another successive theory. Again, one can legitimately ask whether Weyl used philosophy to support his mathematical views or whether at least for the construction of this specific analogy philosophy was one of the ingredients that he needed for the formulation of his theory. To answer this question requires a separate study, but I tried to provide the reader with some hints that will be perhaps useful in pursuing this investigation.

6 Closing Remarks

The aim of this contribution was simply to highlight some philosophical aspects emerging from Weyl's *Raum-Zeit-Materie.* In particular, I wanted to recall his interpretation of relativity theory and underline its importance for current studies in philosophy of time and philosophy of science. I also wanted to point out the presence of Weyl's holism and its characterization before 1927, as well as his view of the structure of scientific theories. Finally, I wanted to stress the importance of considering Weyl's reflection upon the role of geometry and epistemology of science in order to present his unified field theory. One of the major lessons that one can take from Weyl's work is certainly his tendency to overcome dualities in philosophy, such as in the case of elaborating views overcoming the subjective/objective dichotomy or when stating that the process of theory construction is complementary to system analysis. After more than 100 years, Hermann Weyl's work is still stimulating the discussion among physicists, mathematicians and philosophers. A result of which he would certainly be proud.

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